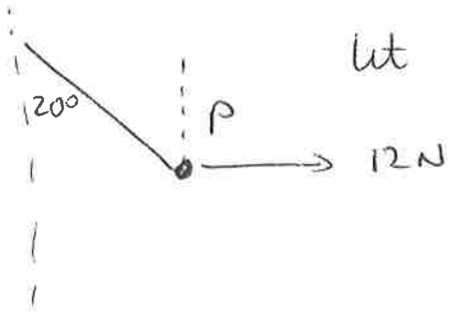
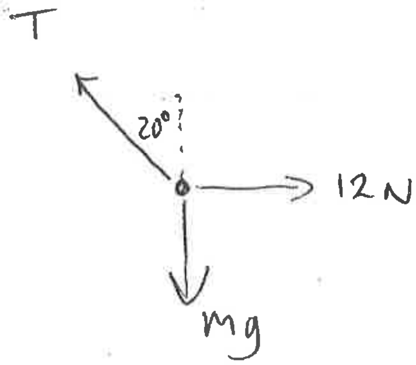
M1 June 2007

(1) cartoon:

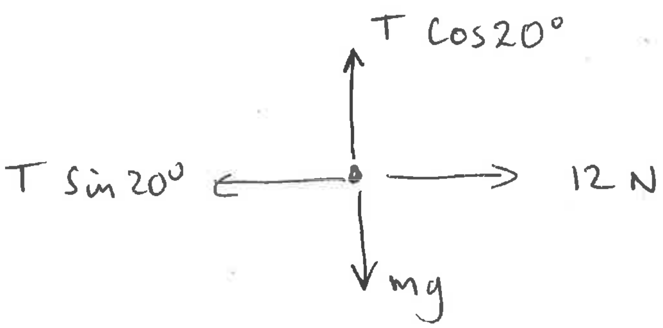


let mass of P be "m"

forces:



resolved forces:



(→): $12 = T\sin 20°$ — (1)

(↑): $T\cos 20° = mg$ — (2)

(a) ① $\Rightarrow$ $T = \dfrac{12}{\sin 20^\circ}$

$\Rightarrow$ $T = 35.0856528$

$\Rightarrow$ $T = 35\text{ N}$ (2 sig. fig.)

(b) ② $\Rightarrow$ $T\cos 20^\circ = mg$

$\cancel{\Rightarrow \quad 35\cos 20^\circ = m \times 9.8}$

$\cancel{\Rightarrow \quad m = \dfrac{35\cos 20}{9.8}}$

$\cancel{m = 3.35604507 4\text{ kg}}$

$\cancel{\Rightarrow \quad m = 3.4\text{ kg (2 sig. fig.)}}$

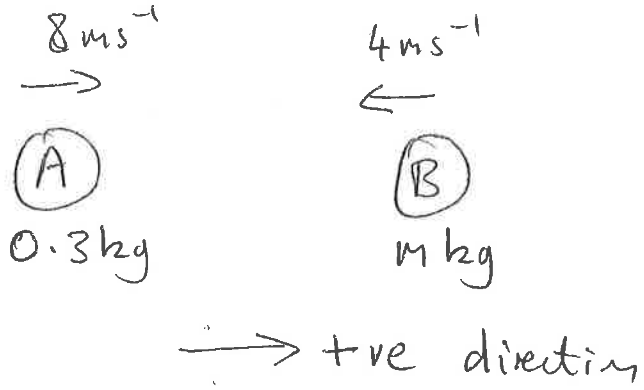
it was the "weight" that was wanted NOT the mass !!!

$\Rightarrow$ $mg = 35.0856528 \times \cos 20^\circ$

$= 32.96972903$

$\Rightarrow$ $mg = 33\text{ N}$ (2 sig. fig)

(2) before:



after: "direction of each is reversed.

conservation of momentum:

$$0.3 \times 8 + m \times (-4) = 0.3 \times (-2) + m \times 2$$

$$\Rightarrow \quad 2.4 - 4m = -0.6 + 2m$$

$$\Rightarrow \quad 3 = 6m$$

$$\Rightarrow \quad m = 0.5 \text{ kg}$$

(a) magnitude of impulse by B on A is magnitude of change of momentum of A

"$I = mv - mu$"

$$I = 0.3 \times 8 - 0.3 \times (-2$$

$$= 2.4 + 0.6$$

$\Rightarrow \quad \underline{\underline{I = 3\,Ns}}$

(b) conservation of momentum as above

$\underline{\underline{m = 0.5\,kg}}$

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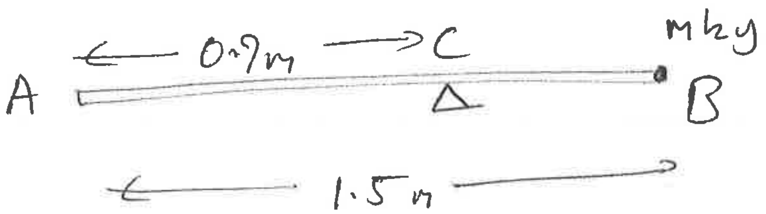
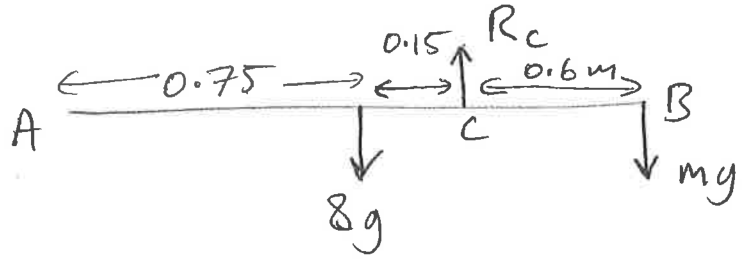
(3)



"uniform"
mass 8 kg
in equilibrium.

forces



(a) ↻ C : $\quad 0.15 \times 8g = 0.6 \times mg$

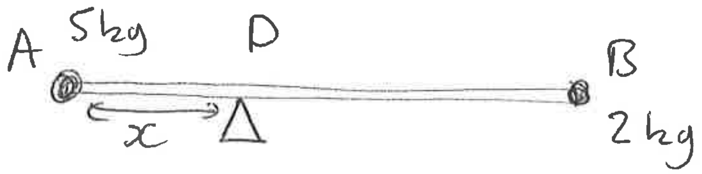
$\Rightarrow \quad 0.15 \times 8 = 0.6 \times m$

$\Rightarrow \quad m = \dfrac{0.15 \times 8}{0.6}$

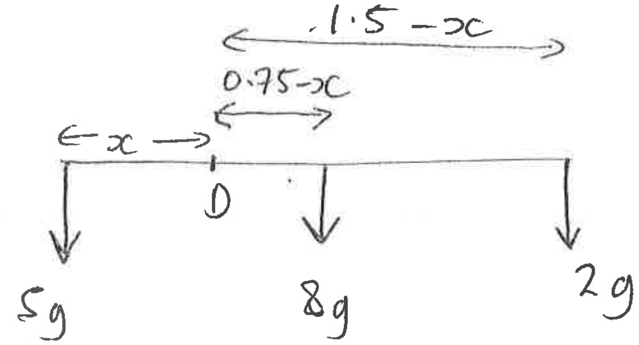
$\underline{\underline{m = 2\,kg}}$ as required.

(b) new diagram

rod 8 kg.



new force diagram :



↺ D:

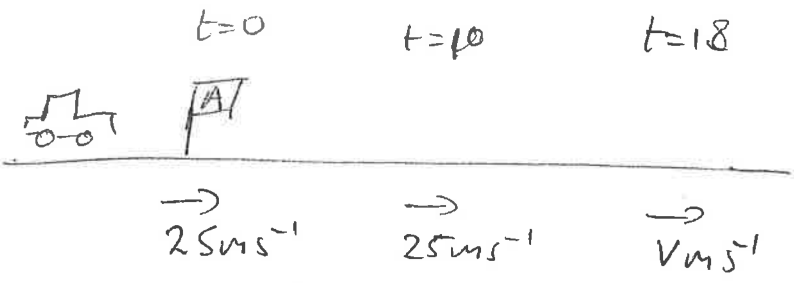
$$x \times 5g = (0.75 - x)\,8g + (1.5 - x) \times 2g$$

$$\Rightarrow \quad 5xg = 6g - 8xg + 3g - 2xg$$

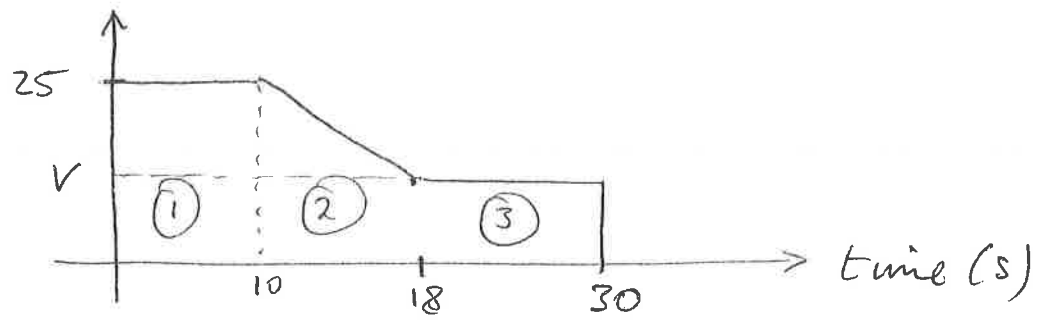
$$\Rightarrow \quad 15xg = 9g$$

$$\Rightarrow \quad x = \frac{3}{5} \quad \text{or} \quad \underline{\underline{0.6\,m}}$$

(4)



(a)



(b) total distance is 526m
split area under into three shapes and add.

$$526 = 10 \times 25 + \tfrac{1}{2}(V+25)\times 8 + 12\times V$$

$$= 250 + 4V + 100 + 12V$$

$$= 350 + 16V$$

hence $176 = 16V$

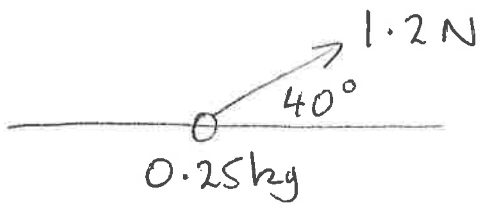
$\Rightarrow$ $V = 11\ ms^{-1}$

(c) $v = u + at \Rightarrow a = \dfrac{v-u}{t}$

$$= \frac{11-25}{8}$$

$$= -\frac{14}{8}\ ms^{-2} \Rightarrow \text{deceleration is } 1.75\ ms^{-2}$$

(5) cartoon:



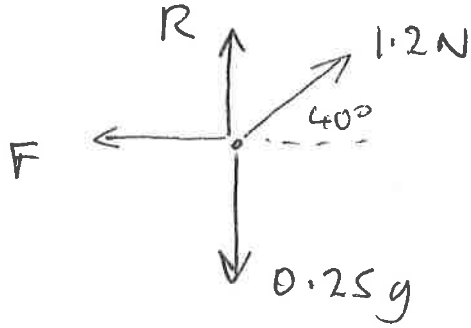
rough

μ

"limiting equilibrium"

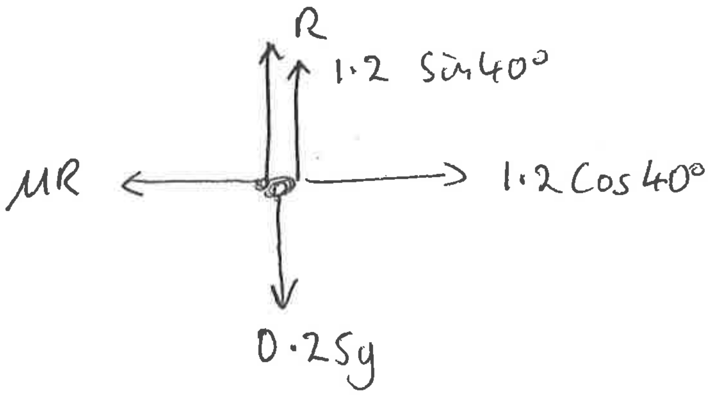
$F = \mu R$

forces:



where $F = \mu R$

resolved forces:



(a) $(\uparrow)$: $R + 1.2\sin 40° = 0.25g$

$\Rightarrow \quad R = 0.25g - 1.2\sin 40°$

$R = 1.678654868$

$\Rightarrow \quad \underline{\underline{R = 1.7\text{ N}}} \quad (2\text{ s.f.})$

(b) $(\rightarrow)$ $\mu R = 1.2 \cos 40°$

$\Rightarrow \mu = \dfrac{1.2 \cos 40°}{1.678654868}$

$\Rightarrow \mu = 0.547603 2752$

$\Rightarrow \underline{\underline{\mu = 0.55}}$ (2 s.f.)

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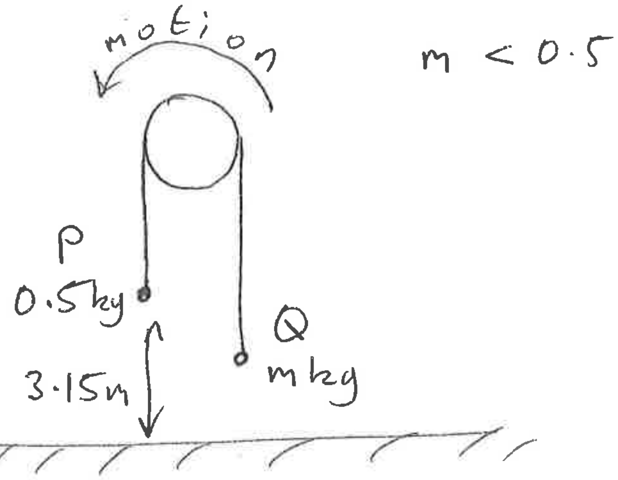
(6) cartoon:



P reaches ground, covering 3.15 m, in 1.5 sec.

(a) ~~$v = u + at \Rightarrow a = v$~~

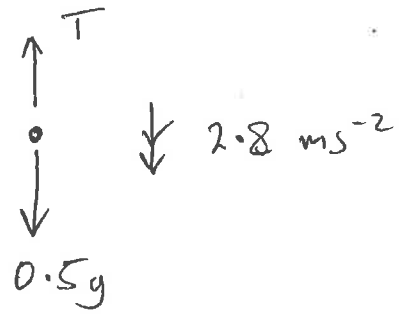
$s = ut + \frac{1}{2}at^2$

$\Rightarrow 3.15 = 0 + \frac{1}{2} \times a \times 1.5^2$

$\Rightarrow a = \dfrac{2 \times 3.15}{1.5^2}$

$\underline{\underline{a = 2.8 \text{ ms}^{-2}}}$

(b) for P :

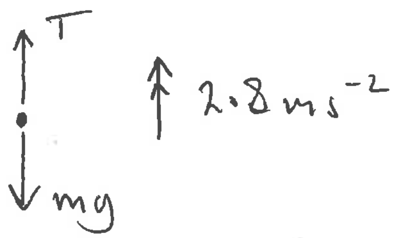


$$N\text{II} \Rightarrow \quad 0{\cdot}5g - T = 0{\cdot}5 \times 2{\cdot}8$$

$$\Rightarrow \quad T = 0{\cdot}5g - 0{\cdot}5 \times 2{\cdot}8$$

$$\Rightarrow \quad \underline{\underline{T = 3{\cdot}5\ \text{N}}}$$

(c) for Q :



$$N\text{II} \Rightarrow \quad T - mg = 2{\cdot}8\,m$$

$$\Rightarrow \quad 3{\cdot}5 - mg = 2{\cdot}8\,m$$

$$\Rightarrow \quad 3{\cdot}5 = 2{\cdot}8\,m + 9{\cdot}8\,m$$

$$\Rightarrow \quad 3{\cdot}5 = 12{\cdot}6\,m$$

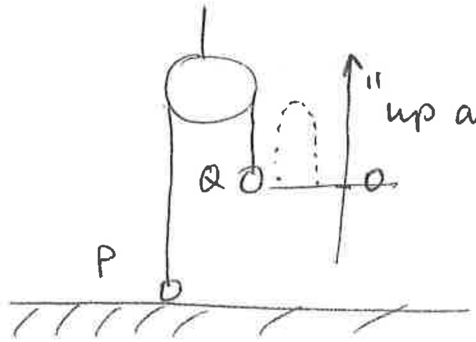
$$\Rightarrow \quad m = \frac{3{\cdot}5}{12{\cdot}6} = \frac{35}{126} = \frac{5}{18}$$

$m = \frac{5}{18}$ as required

(d) string inextensible $\Rightarrow$ acceleration of P down = acceleration of Q up

or motion of the particles is the same.

(c) cartoon :



$a = -9.8 \, ms^{-2}$

NEED VELOCITY AT START OF UP AND DOWN

(velocity) u for up and down

= find velocity of Q in connected particles motion.

ie <u>motion of Q as a connected particle (First Part)</u>

$u = 0, \quad a = 2.8 \, ms^{-2}, \quad s = 3.15 \, m$

$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times 2.8 \times 3.15$

$\Rightarrow v^2 = 17.64$

<u>motion of Q as "up and down"</u>

$u = \sqrt{17.64}$, time to return to zero point.

$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = \sqrt{17.64} \times t + \frac{1}{2} \times (-9.8) t^2$

$\Rightarrow \quad 0 = \sqrt{17.64}\, t - 4.9\, t^2$

$\Rightarrow \quad 0 = t\left(\sqrt{17.64} - 4.9t\right)$

and $t = 0$ at start of up and down

or $t = \dfrac{\sqrt{17.64}}{4.9} = 0.857142857$

$\underline{\underline{t = 0.86 \text{ sec}}}$ (2 s.f.)

⑦ At noon:

boat B: $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

NB 1200 to 1430 is 2 hrs 30 min $= 2.5$ hours

At 1430

boat B: $\overrightarrow{OB} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$

(a) velocity of B is constant, hence,

$$\underset{\sim}{v} = \frac{\text{final displ.} - \text{initial displ.}}{\text{time taken}}$$

$$= \frac{\begin{pmatrix} 8 \\ 11 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{2.5} = \frac{\begin{pmatrix} 5 \\ 15 \end{pmatrix}}{2.5} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

velocity of B is $\begin{pmatrix} 2 \\ 6 \end{pmatrix} \text{ms}^{-1} = \underline{\underline{2\underset{\sim}{i} + 6\underset{\sim}{j} \text{ ms}^{-1}}}$

(b) Position of B at t hours after noon is $\underset{\sim}{b}$ km

$\underset{\sim}{b}$ = start position + t × velocity

$$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$\Rightarrow \underset{\sim}{b} = \begin{pmatrix} 3 + 2t \\ 6t - 4 \end{pmatrix}$ km or $(3+2t)\underset{\sim}{i} + (6t-4)\underset{\sim}{j}$

Boat C is at $\underset{\sim}{c} = \begin{pmatrix} -9+6t \\ 20+dt \end{pmatrix}$

(c) C intercepts B if and only if there exists a t such that,

$$\begin{pmatrix} 3+2t \\ 6t-4 \end{pmatrix} = \begin{pmatrix} -9+6t \\ 20+dt \end{pmatrix}$$

ie $3+2t = -9+6t$ — ①

and

$6t-4 = 20+dt$ — ②

① $\Rightarrow$ $12 = 4t$ and $t = 3$

so ② $\Rightarrow$ $6\times3-4 = 20+d\times3$

$\Rightarrow$ $14 = 20+3d$

$\Rightarrow$ $-6 = 3d$

$\Rightarrow$ $\underline{\underline{d = -2}}$

(d) recognise that velocity of C is $6\underset{\sim}{i} + d\underset{\sim}{j}$

ie $6\underset{\sim}{i} - 2\underset{\sim}{j}$

hence <u>speed</u> of C is $\sqrt{6^2+(-2)^2} = \sqrt{40}$

now speed of B is

$$\sqrt{2^2+6^2} = \sqrt{40}$$

hence B and C have same speed.

0